basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2022

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages and 1 information sheet.

MATHE2

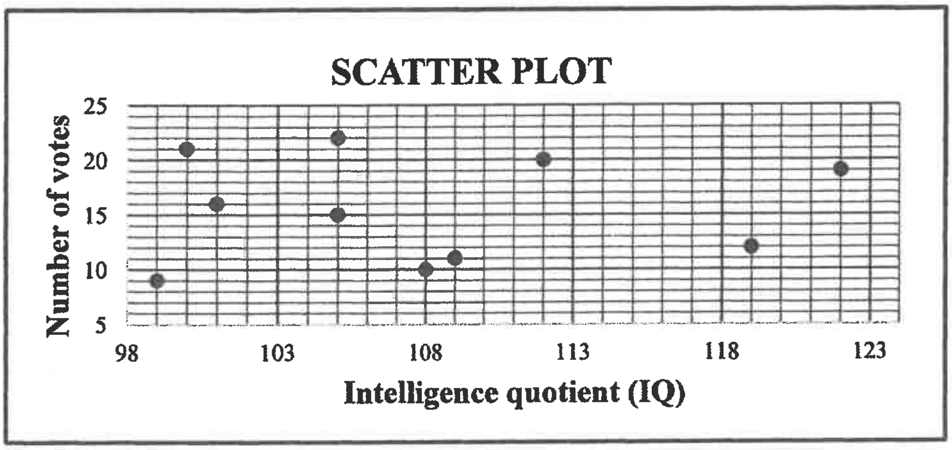
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The matric class of a certain high school had to vote for the chairperson of the RCL (representative council of learners). The scatter plot below shows the IQ (intelligence quotient) of the 10 learners who received the most votes and the number of votes that they received.



Before the election, the popularity of each of these ten learners was established and a popularity score (out of a 100) was assigned to each. The popularity scores and the number of votes of the same 10 learners who received the most votes are shown in the table below.

Popularity score (x)	32	89	35	82	50	59	81	40	79	65
Number of votes (y)	9	22	10	21	11	15	20	12	19	16

1.1 Calculate the:

1.1.1 Mean number of votes that these 10 learners received (2)

1.1.2 Standard deviation of the number of votes that these 10 learners received (1)

1.2 The learners who received fewer votes than one standard deviation below the mean were not invited for an interview. How many learners were invited? (2)

1.3 Determine the equation of the least squares regression line for the data given in the table. (3)

1.4 Predict the number of votes that a learner with a popularity score of 72 will receive. (2)

1.5 Using the scatter plot and table above, provide a reason why:

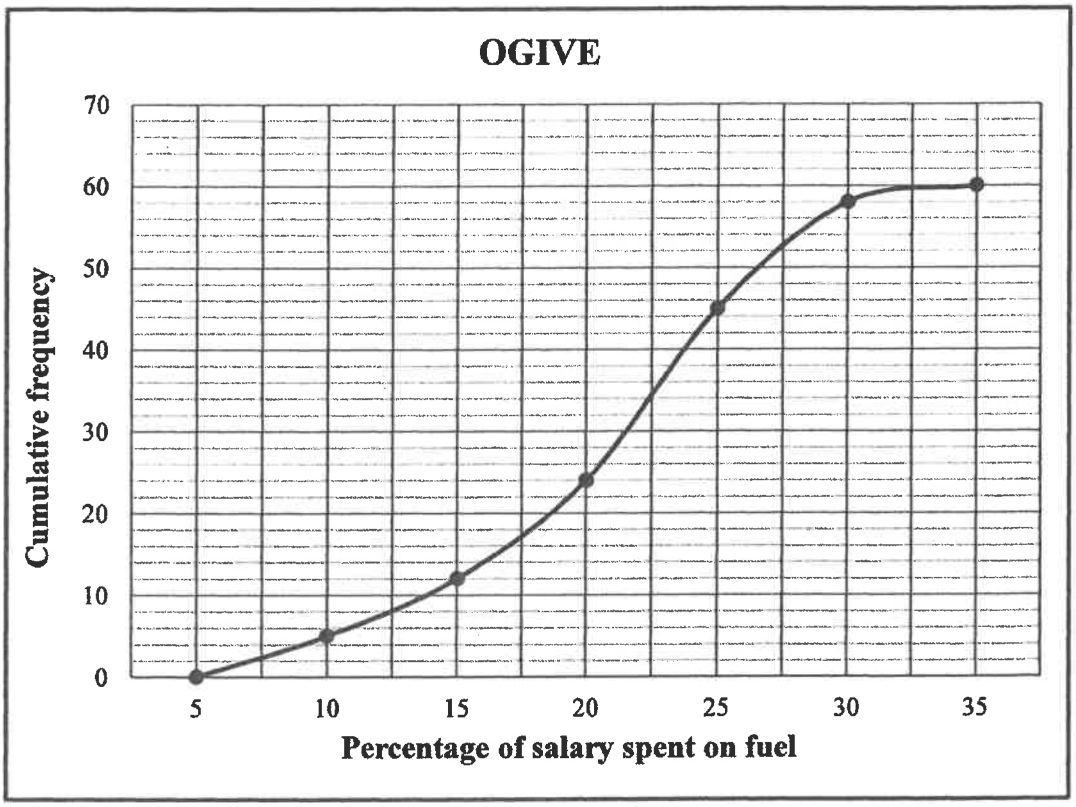
1.5.1 IQ is not a good indicator of the number of votes that a learner could receive (1)

1.5.2 The prediction in QUESTION 1.4 is reliable (1)

[12]

QUESTION 2

A company conducted research among all its employees on what percentage of their monthly salary was spent on fuel in a particular month. The data is represented in the ogive (cumulative frequency graph) below.



2.1 How many people are employed at this company? (1)

2.2 Write down the modal class of the data. (1)

2.3 How many employees spent more than 22,5% of their monthly salary on fuel? (2)

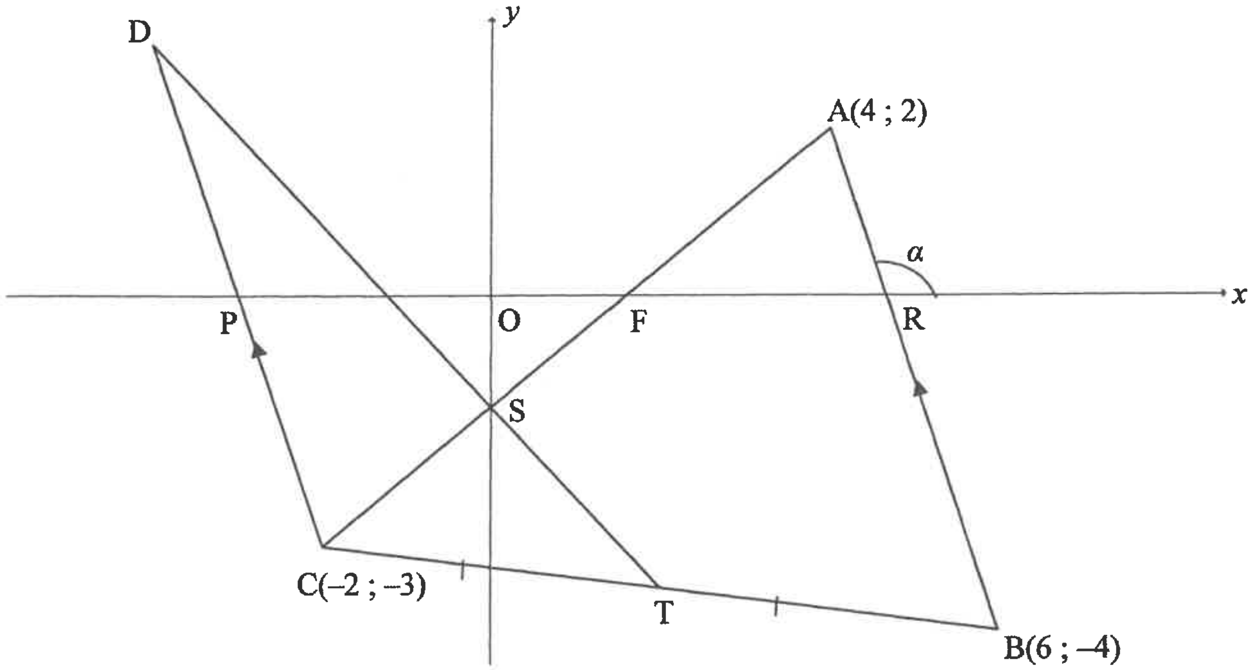
2.4 An employee spent R2 400 of his salary on fuel in that particular month. Determine the monthly salary of this employee if he spends 7% of his salary on fuel. (2)

2.5 The monthly salaries of these employees remains constant and the number of litres of fuel used in each month also remains constant. If the fuel price increases from R21,43 per litre to R22,79 per litre at the beginning of the next month, how will the above ogive change? (2)

[8]

QUESTION 3

In the diagram, A(4 ; 2), B(6 ; –4) and C(–2 ; –3) are vertices of ΔABC. T is the midpoint of CB. The equation of line AC is $5x-6y=8$. The angle of inclination of AB is α. ΔDCT is drawn such that CD || BA. The lines AC and DT intersect at S, the y-intercept of AC. P, F and R are the x-intercepts of DC, AC and AB respectively.



3.1 Calculate the:

3.1.1 Gradient of AB (2)

3.1.2 Size of α (2)

3.1.3 Coordinates of T (2)

3.1.4 Coordinates of S (2)

3.2 Determine the equation of CD in the form $y = mx + c$. (3)

3.3 Calculate the:

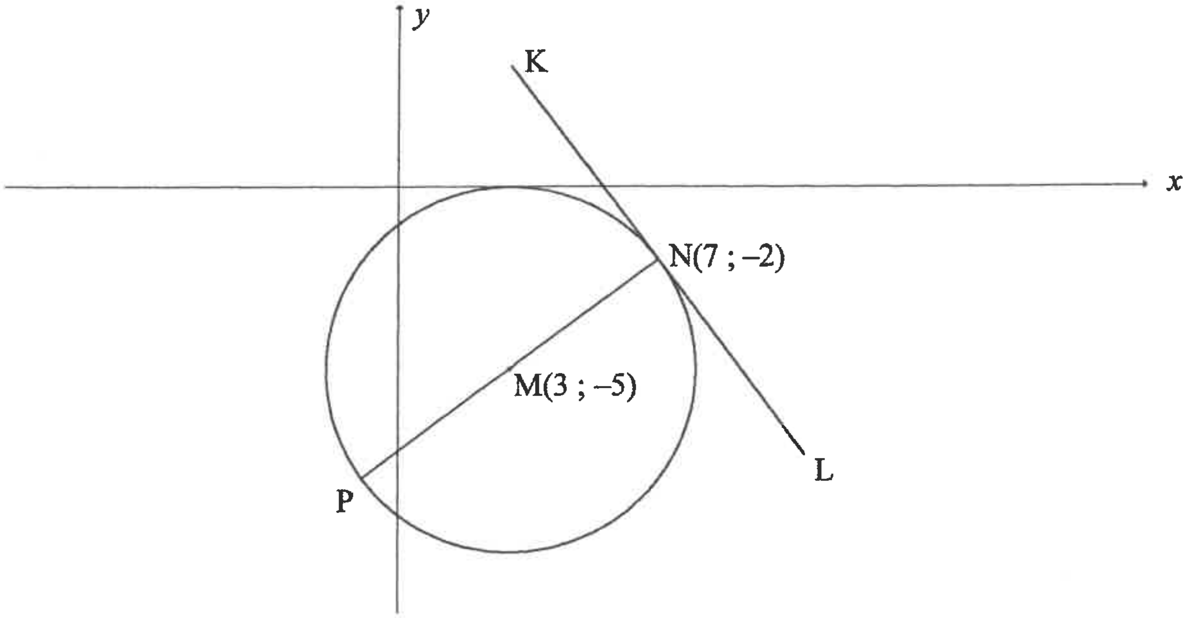
3.3.1 Size of $D\hat{C}A$ (4)

3.3.2 Area of POSC (5)

[20]

QUESTION 4

In the diagram, M(3 ; –5) is the centre of the circle having PN as its diameter. KL is a tangent to the circle at N(7 ; –2).



4.1 Calculate the coordinates of P. (2)

4.2 Determine the equation of:

4.2.1 The circle in the form $(x-a)^2+(y-b)^2=r^2$ (3)

4.2.2 KL in the form $y=mx+c$ (5)

4.3 For which values of k will $y=-\frac{4}{3}x+k$ be a secant to the circle? (4)

4.4 Points A(t ; t) and B are not shown on the diagram.

From point A, another tangent is drawn to touch the circle with centre M at B.

4.4.1 Show that the length of tangent AB is given by $\sqrt{2t^2+4t+9}$. (2)

4.4.2 Determine the minimum length of AB. (4)

[20]

QUESTION 5

5.1 Given that $\sqrt{13}\sin x + 3 = 0$, where $x \in (0°\,;\,90°)$.

Without using a calculator, determine the value of:

5.1.1 $\sin(360° + x)$ (2)

5.1.2 $\tan x$ (3)

5.1.3 $\cos(180° + x)$ (2)

5.2 Determine the value of the following expression, **without using a calculator**:

$$\frac{\cos(90° + \theta)}{\sin(\theta - 180°) + 3\sin(-\theta)}$$ (5)

5.3 Determine the general solution of the following equation:

$(\cos x + 2\sin x)(3\sin 2x - 1) = 0$ (6)

5.4 Given the identity: $\cos(x + y).\cos(x - y) = 1 - \sin^2 x - \sin^2 y$

5.4.1 Prove the identity. (4)

5.4.2 Hence, determine the value of $1 - \sin^2 45° - \sin^2 15°$, **without using a calculator.** (3)

5.5 Consider the trigonometric expression: $16\sin x.\cos^3 x - 8\sin x.\cos x$

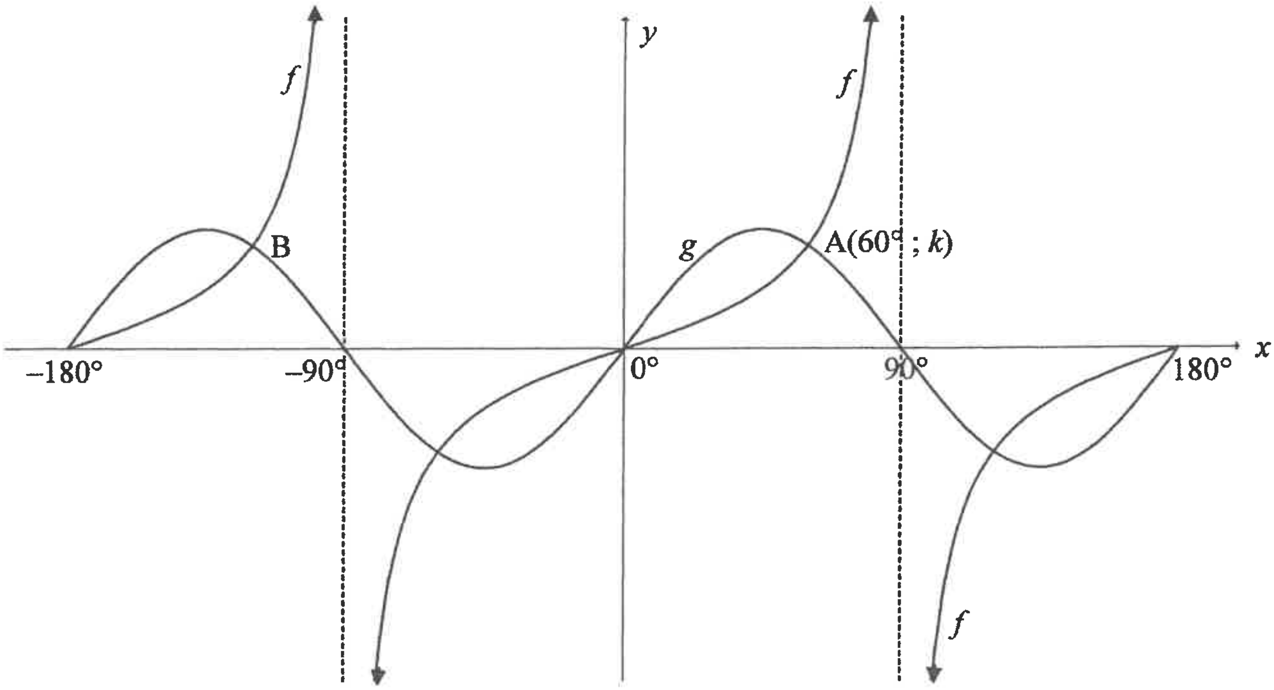
5.5.1 Rewrite the expression as a single trigonometric ratio. (4)

5.5.2 For which value of x in the interval $x \in [0°\,;\,90°]$ will $16\sin x.\cos^3 x - 8\sin x.\cos x$ have its minimum value? (1)

[30]

QUESTION 6

In the diagram below, the graphs of $f(x) = \tan x$ and $g(x) = 2\sin 2x$ are drawn for the interval $x \in [-180° ; 180°]$ A(60° ; k) and B are two points of intersection of f and g.



6.1 Write down the period of g. (1)

6.2 Calculate the:

6.2.1 Value of k (1)

6.2.2 Coordinates of B (1)

6.3 Write down the range of $2g(x)$. (2)

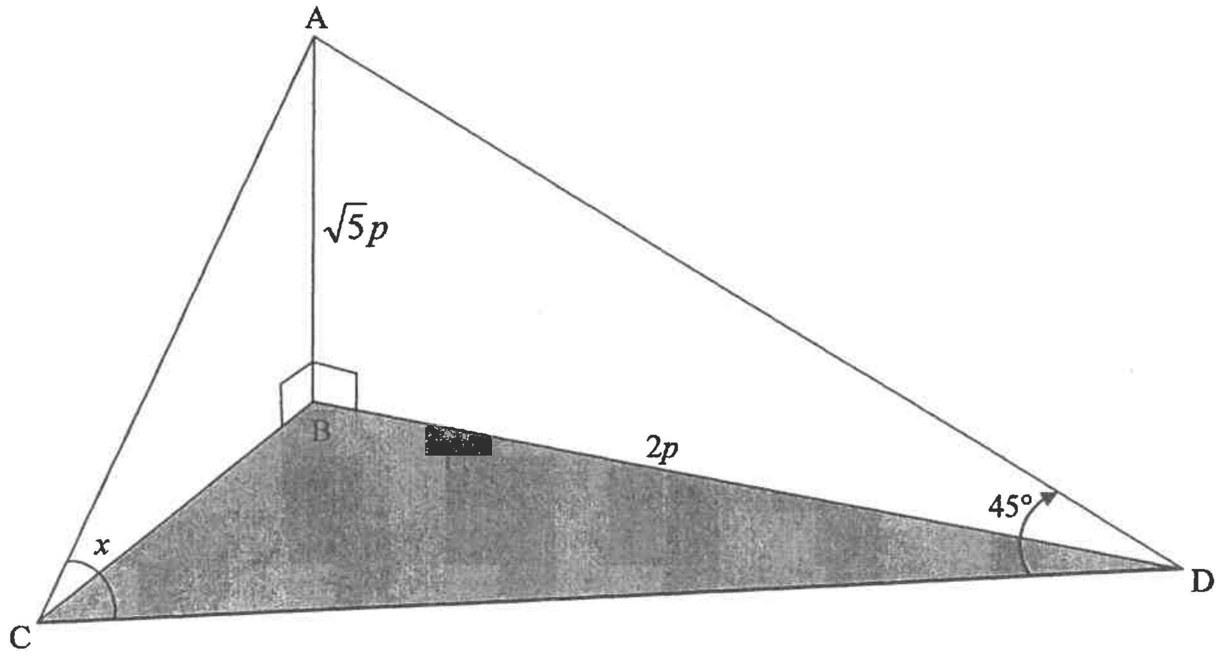
6.4 For which values of x will $g(x+5°) - f(x+5°) \leq 0$ in the interval $x \in [-90° ; 0°]$? (2)

6.5 Determine the values of p for which $\sin x.\cos x = p$ will have exactly two real roots in the interval $x \in [-180° ; 180°]$. (3)

[10]

QUESTION 7

AB is a vertical flagpole that is $\sqrt{5}p$ metres long. AC and AD are two cables anchoring the flagpole. B, C and D are in the same horizontal plane.
$BD = 2p$ metres, $A\hat{C}D = x$ and $A\hat{D}C = 45°$.



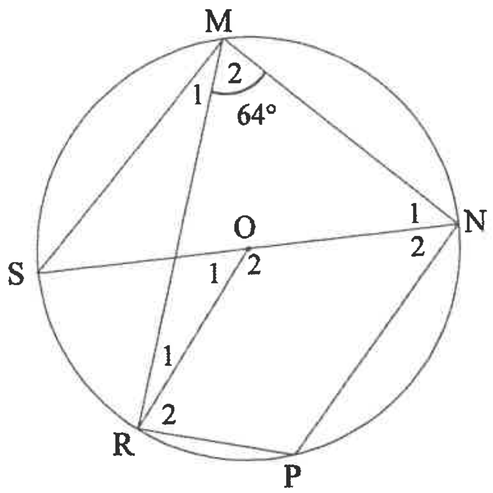
7.1 Determine the length of AD in terms of p. (2)

7.2 Show that the length of $CD = \dfrac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}$. (5)

7.3 If it is further given that $p = 10$ and $x = 110°$, calculate the area of ΔADC. (3)
[10]

QUESTION 8

8.1 In the diagram, O is the centre of the circle. MNPR is a cyclic quadrilateral and SN is a diameter of the circle. Chord MS and radius OR are drawn. $\hat{M}_2 = 64°$.



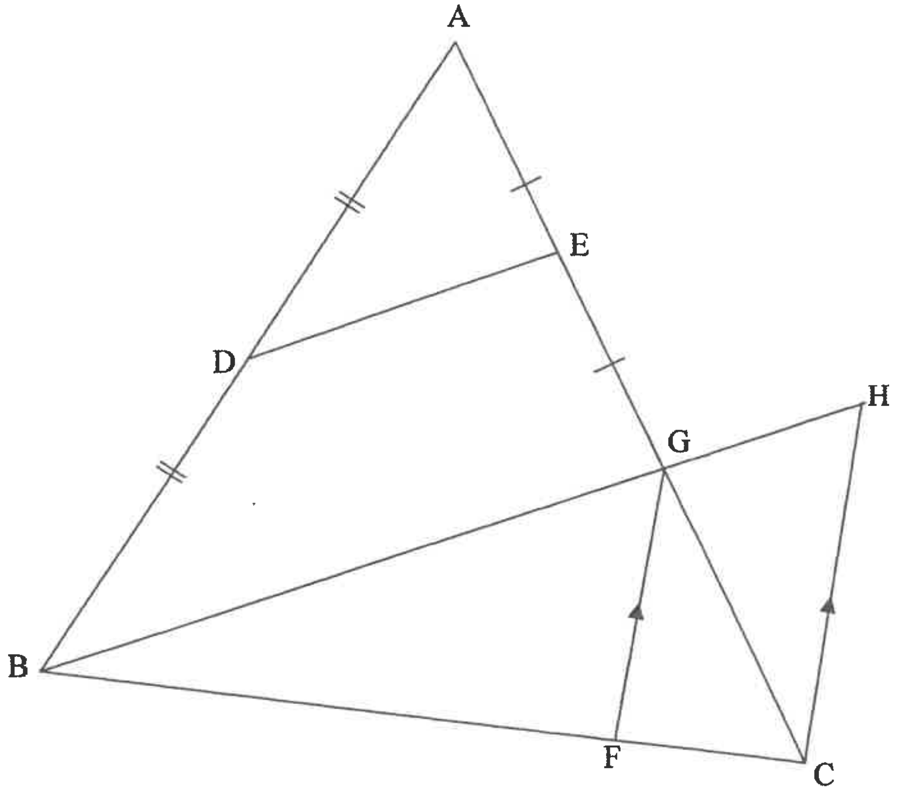
Determine, giving reasons, the size of the following angles:

8.1.1 $\hat{P}$ (2)

8.1.2 $\hat{M}_1$ (2)

8.1.3 $\hat{O}_1$ (2)

8.2 In the diagram, ΔABG is drawn. D and E are midpoints of AB and AG respectively. AG and BG are produced to C and H respectively. F is a point on BC such that FG || CH.



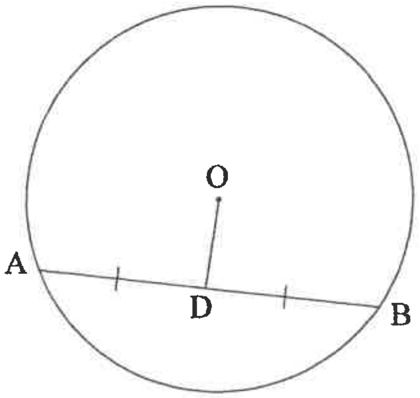
8.2.1 Give a reason why DE || BH. (1)

8.2.2 If it is further given that $\frac{FC}{BF}=\frac{1}{4}$, DE = $3x-1$ and GH = $x+1$, calculate, giving reasons, the value of x. (6)

[13]

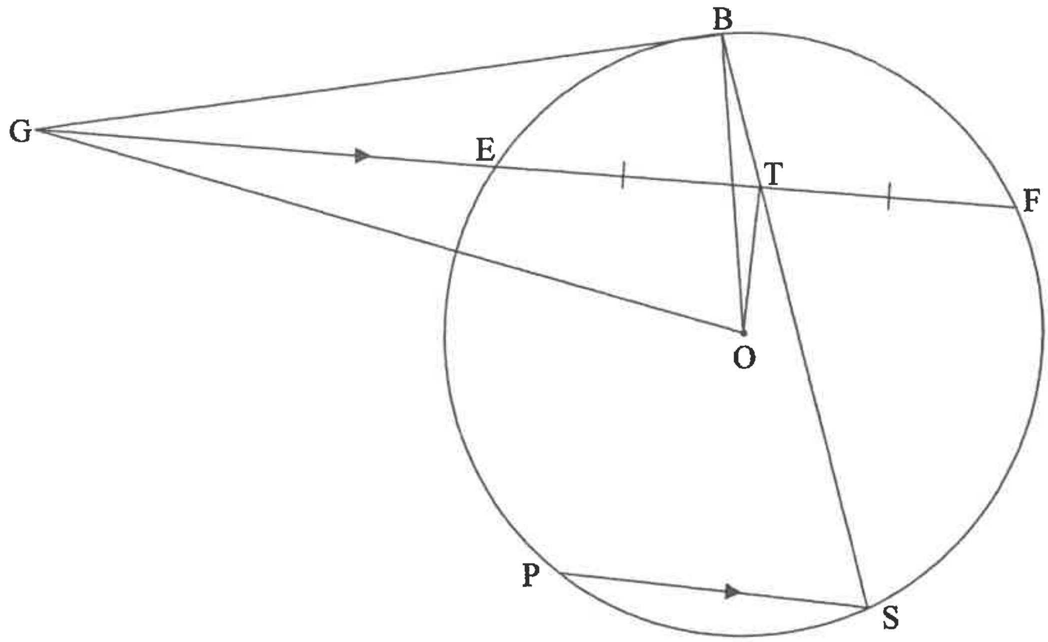
QUESTION 9

9.1 In the diagram, O is the centre of a circle. OD bisects chord AB.



Prove the theorem that states that the line from the centre of a circle that bisects a chord is perpendicular to the chord, i.e. $OD \perp AB$. (5)

9.2 In the diagram, E, B, F, S and P are points on the circle centred at O. GB is a tangent to the circle at B. FE is produced to meet the tangent at G. OT is drawn such that T is the midpoint of EF. GO and BO are drawn. BS is drawn through T. PS || GF.



Prove, giving reasons, that:

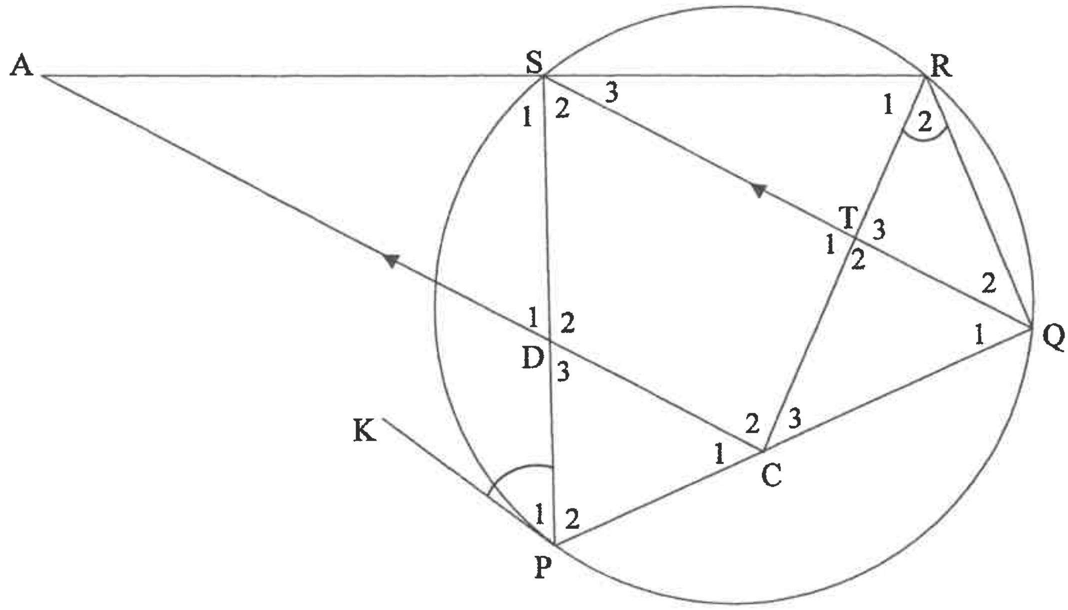
9.2.1 OTBG is a cyclic quadrilateral (5)

9.2.2 $G\hat{O}B = \hat{S}$ (4)

[14]

QUESTION 10

In the diagram, PQRS is a cyclic quadrilateral. KP is a tangent to the circle at P. C and D are points on chords PQ and PS respectively and CD produced meets RS produced at A. CA || QS. RC is drawn. $\hat{P}_1 = \hat{R}_2$.



Prove, giving reasons, that:

10.1 $\hat{S}_1 = \hat{T}_2$ (4)

10.2 $\frac{AD}{AR} = \frac{AS}{AC}$ (5)

10.3 $AC \times SD = AR \times TC$ (4)

[13]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$